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UNIDIMENSIONAL MODEL FOR PIPE FLOW OF A GAS MIXTURE ALLOWING FOR CONDENSATION

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Condensation kinetics is described with allowance for the dependence of the temperature of the phase transformation on the pressure and concentration of the condensing gas at the phase boundary.

We will examine the turbulent axisymmetric flow of a binary gas mixture in a pipe. Let the flow conditions be such that one of the components of the mixture condenses on the pipe surface, forming a film of liquid condensate of a thickness which increases with time.

We will describe the flow of the condensing gas with a two-layer model of turbulent flow — a laminar sublayer close to the surface of the condensate layer and a turbulent core in the remaining part of the flow. Intensive turbulent transport of the substance in the flow core makes the temperature, pressure, density, velocity, and concentration of the condensing gas practically constant across the pipe. Thus, we will use cross-sectional-mean values of these parameters to describe transport processes in the flow core, with the parameters changing only with time and station (from cross section to cross section). In the laminar sublayer between the flow core and the surface of the condensate film, the parameter values change from the values in the core to the values on the surface.

The liquid in the film will be assumed stationary in order to simplify the construction of a one-dimensional model of the process, here retaining all of the essential features of the flow.

The unidimensional equations for the mean (across the gas flow) quantities have the form

$$\frac{\partial \rho S_w}{\partial t} + \frac{\partial \rho u S_w}{\partial x} = \dot{M} S_w,$$

$$\dot{M} = -\frac{\Pi_w}{S_w}\left[\rho v_w - \rho u_w \frac{\partial R_w}{\partial x} - \rho \frac{\partial R_w}{\partial t}\right],$$

$$\frac{\partial \rho u S_w}{\partial t} + \frac{\partial \rho u^2 S_w}{\partial x} = -S_w \frac{\partial p}{\partial x} + \tau_w \Pi_w + u_w \dot{M} S_w - \rho S_w g \sin \alpha^*,$$

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$$\frac{\partial \rho\theta S_w}{\partial t} + \frac{\partial \rho u\theta S_w}{\partial x} = S_w \dot{M} c_w - \Pi_w \rho j_w,$$

$$\frac{\partial \rho H S_w}{\partial t} + \frac{\partial \rho u H S_w}{\partial x} - S_w \frac{\partial p}{\partial t} = \dot{M} S_w H_w + \tau_w u_w \Pi_w - q_w \Pi_w - \rho g u \sin\alpha^*,$$

$$H = \mathrm{E} + \frac{p}{\rho} + \frac{|\mathfrak{M}|^2}{2}. \tag{1}$$

These equations can be derived on the basis of integral balance equations for a gas particle the lateral surface of which at any moment of time coincides with the surface of the film and the end surfaces of which coincide with arbitrary sections of the gas flow. Here, it should be considered that the normal components of the velocities of the particle and the film surface are not equal [1].

At the phase boundary — where several of the local parameters of the medium undergo a discontinuous change — the following conditions of conservation of the total mass and energy of the flow and the mass of the condensing component are satisfied:

$$[\rho(\mathfrak{W}_n - D_n)] = 0,$$

$$[\rho h(\mathfrak{W}_n - D_n)] = -[q_n],$$

$$[\rho c(\mathfrak{W}_n - D_n)] = -[j_n],$$

$$[\mathfrak{W}_\tau] = 0, \quad [p_{n\tau}] = 0,$$

$$h = \varepsilon + \frac{p}{\rho}. \tag{2}$$

Here, the first condition of total-mass balance serves to determine the radial component of velocity v_W at the boundary, since u_W is considered to be zero.

Allowing for the first equation of (2) and the fact that $[h] = L$, we reduce the condition of energy balance to the form

$$\rho(\mathfrak{W}_n - D_n) L = -[q_n]. \tag{3}$$

We will assume the temperature of the phase boundary to be equal to the condensation temperature, which is dependent on the partial pressure of the condensing gas p_{1W}. To determine the latter, we have the following from the Clausius–Clapeyron conditions at L = const

$$p_{1w} = p_* \exp\left(-\frac{T_*}{T_w}\right). \tag{4}$$

We will write the equation of state of the gas mixture as:

$$p = p(\rho, T, \theta). \tag{5}$$

If we assume that the components of the mixture are perfect gases, then the following equations of state are satisfied for each of them:

$$p_1 = \frac{R}{m_1}\rho_1 T, \quad p_2 = \frac{R}{m_2}\rho_2 T. \tag{6}$$

Then the equation of state (5) for the mixture is

$$p = \frac{RT}{m_2}\rho\left[1 + \left(\frac{m_2}{m_1} - 1\right)\theta\right]. \tag{7}$$

The partial pressure of the condensing gas p_{1W} can be expressed through the total pressure p and c_W and substituted into (4):

$$\frac{p}{p_*} = \left[1 - \frac{m_1}{m_2}\left(1 - \frac{1}{c_w}\right)\right]\exp\left(-\frac{T_*}{T_w}\right). \tag{8}$$

The unidimensional equations (1) and phase-boundary conditions (2) contain local quantities — flows of mass, energy, momentum, velocity, etc. on the film surface. To close the system of equations, it is to indicate the relation between these quantities and the governing parameters of the unidimensional model. Such relations cannot be found within the framework of a unidimensional model and involve additional considerations for their determination. Here, either experimental results can be used, or simpler, three-dimensional models can be employed to describe transfer processes near the phase boundary, these models allowing thorough mathematical investigation of the problem. Also, these two approaches can be combined.

Based on dimensional considerations, we can write the frictional stress and the heat and mass flow on the film surface in the form

$$\tau_w = -\zeta \frac{\rho u^2}{2}, \quad j_n = \beta(\theta - c_w), \quad q_n = \alpha(T - T_w). \tag{9}$$

Now the only difficulty in closing the unidimensional equations lies in finding the dependence of the coefficients ζ, β, and α on the parameters of the unidimensional model. Here, we can use the results of experimental studies involving a two-phase annular flow [2]. Let appropriate relations for ζ, β, and α be chosen. There remains yet one more complication in unidimensionally describing the process in question. It is necessary to indicate the temperature and concentration at the moment of formation of the condensate film. This problem arises in connection with the fact that the phase transition temperature depends not only on the pressure in the system, but also on the concentration of the condensing gas in the flow.

The following approach is suggested for solving this problem. During the initial part of the life of the film, the thermal and diffusion layers are much thinner than the laminar sublayer. During this period, transfer processes occur deep within this sublayer. They can therefore be described by formulating a Stefan-type problem allowing for heat and mass transfer.

Let us examine the half-space $y > 0$, filled at the moment $t = 0$ with a quiescent mixture of gases with the temperature T_0 at a pressure p. The solid boundary of the half-space has a temperature T_b which is lower than the condensation temperature of one of the components under the conditions prevailing in the external region. In such a case, a layer of condensate is formed on the solid boundary. The coordinate of the phase boundary will be designated through $y_0(t)$.

We will write the equations of motion, energy, and diffusion for the condensing component in the gas:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial y^2}, \quad v = v(t, y), \quad y > y_0(t), \tag{10}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \varkappa \frac{\partial^2 T}{\partial y^2}, \quad T = T(t, y), \quad y > y_0(t), \tag{11}$$

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2}, \quad c = c(t, y), \quad y > y_0(t). \tag{12}$$

The equation of heat flow to the condensate layer has the form

$$\frac{\partial T_l}{\partial t} = \varkappa_l \frac{\partial^2 T_l}{\partial y^2}; \quad T_l = T_l(t, y), \quad y < y_0(t). \tag{13}$$

The conditions on the phase boundary (similar to the conditions (2)):

$$\rho(v_w - \dot{y}_0) = -\rho_l \dot{y}_0, \quad y = y_0(t), \quad t > 0, \tag{14}$$

$$\rho_l \dot{y}_0 L = -\left(\lambda \frac{\partial T}{\partial y}\right)_w + \left(\lambda_l \frac{\partial T_l}{\partial y}\right)_w, \quad y = y_0(t), \quad t > 0, \tag{15}$$

$$\rho c_w (v_w - \dot{y}_0) - \rho D \left(\frac{\partial c}{\partial y}\right)_w = -\rho_l \dot{y}_0, \quad y = y_0(t), \quad t > 0, \tag{16}$$

where c_w is determined by Eq. (8).

At $y = \infty$

$$v = 0,\ T = T_0,\ c = c_0, \tag{17}$$

at y = 0

$$T = T_b.$$

This system of equations differs from the system which describes the process without allowance for mass transfer in that it has an additional equation for the concentration of the condensing gas and an additional condition (16) for the phase boundary. Also, it is assumed that the phase transition temperature depends on the concentration of the gas in the vicinity of the phase boundary.

We will introduce the similarity variable $\eta = y/2\sqrt{\nu t}$ and change over to dimensionless variables:

$$\beta = \frac{y_0}{2\sqrt{\nu t}},\quad f(\eta) = \frac{v}{2}\Big/\sqrt{\frac{\nu}{\pi t}},\quad \varphi(\eta) = \frac{T}{T_0},$$

$$\psi(\eta) = \frac{T_l}{T_b},\quad \omega(\eta) = \frac{c}{c_0},\quad \varepsilon = \frac{\rho_l - \rho}{\rho},$$

$$\mathrm{Pr} = \frac{\nu}{\varkappa},\quad \mathrm{Sc} = \frac{\nu}{D},\quad \mathrm{Pr}_l = \frac{\nu}{\varkappa_l}. \tag{18}$$

Then the equations (10)-(13) and boundary conditions (17) are written thus:

$$f'' + \frac{4}{\sqrt{\pi}} f'f - 2(f + \eta f') = 0,\ f = f(\eta),$$

$$\beta < \eta < \infty,\ f(\infty) = 0,\ f(\beta) = -\frac{\sqrt{\pi}}{2}\varepsilon\beta,$$

$$\varphi'' + \left(2\mathrm{Pr}\,\eta - \frac{4\mathrm{Pr}}{\sqrt{\pi}} f(\eta)\right)\varphi' = 0,\ \varphi = \varphi(\eta),$$

$$\beta < \eta < \infty,\ \varphi(\infty) = 1,\ \varphi(\beta) = T_w/T_0,$$

$$\omega'' + \left(2\mathrm{Sc}\,\eta - \frac{4\mathrm{Sc}}{\sqrt{\pi}} f(\eta)\right)\omega' = 0,$$

$$\omega = \omega(\eta),\ \beta < \eta < \infty,\ \omega(\infty) = 1,\ \omega(\beta) = c_w/c_0,$$

$$\psi'' + 2\mathrm{Pr}_l\eta\psi' = 0,$$

$$\psi = \psi(\eta),\ 0 < \eta < \beta,\ \psi(0) = 1,\ \psi(\beta) = T_w/T_b. \tag{19}$$

Integrating these equations, we obtain

$$f(\eta) = \frac{\varepsilon\beta \exp[-(\eta^2 - \beta^2)]}{\frac{2}{\sqrt{\pi}} + \varepsilon\beta \exp\beta^2[\operatorname{erf}\eta - \operatorname{erf}\beta]},\ \beta < \eta < \infty,$$

$$\varphi(\eta) = 1 + \left(\frac{T_w}{T_0} - 1\right)\frac{I(\eta,\ \mathrm{Pr})}{I(\beta,\ \mathrm{Pr})},\ \beta < \eta < \infty,$$

$$\omega(\eta) = 1 + \left(\frac{c_w}{c_0} - 1\right)\frac{I(\eta,\ \mathrm{Sc})}{I(\beta,\ \mathrm{Sc})},\ \beta < \eta < \infty,$$

$$\psi(\eta) = 1 + \left(\frac{T_w}{T_b} - 1\right)\frac{\operatorname{erf}(\sqrt{\mathrm{Pr}_l}\,\eta)}{\operatorname{erf}(\sqrt{\mathrm{Pr}_l}\,\beta)},\ 0 < \eta < \beta, \tag{20}$$

where

$$I(\eta,\ \mathrm{Pr}) = \int_\eta^\infty \frac{\exp(-\mathrm{Pr}\,\xi^2)\,d\xi}{\left[\frac{2}{\sqrt{\pi}} + \varepsilon\beta\exp\beta^2(\operatorname{erf}\xi - \operatorname{erf}\beta)\right]^{2\mathrm{Pr}}}.$$

Substituting (20) into the phase-boundary conditions (15) and (16) (condition (14), used to find v_w and determine the function $f(\eta)$), we obtain

$$\beta\delta = \left(\frac{c_w}{c_0} - 1\right)\frac{\exp(-\mathrm{Sc}\,\beta^2)}{\left(\frac{2}{\sqrt{\pi}}\right)^{2\mathrm{Sc}} I(\beta,\ \mathrm{Sc})},$$

$$\sigma\beta = \left(\frac{T_w}{T_\mathrm{b}} - 1\right)\frac{\exp(-\mathrm{Pr}_l\beta^2)}{\int\limits_0^\beta \exp(-\mathrm{Pr}_l\xi^2)\,d\xi} + \left(\frac{T_w}{T_0} - 1\right)\frac{\gamma\exp(-\mathrm{Pr}\,\beta^2)}{\left(\frac{2}{\sqrt{\pi}}\right)^{2\mathrm{Pr}} I(\beta,\ \mathrm{Pr})}, \qquad (21)$$

where

$$\gamma = \frac{\lambda T_0}{\lambda_l T_\mathrm{b}},\quad \sigma = \frac{2\rho_l L\nu}{\lambda_l T_\mathrm{b}},\quad \delta = \frac{2\rho_l\,\mathrm{Sc}}{\rho c_0}.$$

Equations (8) and (21) represent a closed system of transcendental equations for determining the temperature of the phase boundary T_w, the concentration of the condensing component at the boundary c_w, and the dimensionless velocity of the boundary β. The parameters T_w and c_w, serving as initial conditions in the solution of system (1), are found by solving these equations.

As an example, let us look at the results of a numerical solution of system (21) in the case of an air–steam mixture: $p = 4.0$ MPa, $c_0 = 0.003$, $T_0 = 340$°K, $T_b = 280$°K, $\beta = 7.27\cdot 10^{-4}$, $c_w = 2.53\cdot 10^{-4}$, and $T_w = 280.002$°K.

In this example, the temperature on the surface of the layer turns out to be very close to the temperature of the wall T_b and different from the temperature in the gas flow. The latter causes the mass flow to be directed toward the film. This fact is not considered in models in which the temperature of the phase transition is either constant or is dependent only on the pressure in the system.

Such are the basic features of formulating the problem of the condensation of one of the gases in a mixture on the surface of a pipe, making it possible to reduce the description of the process to solving system of unidimensional equations (1) with allowance for conditions on the surface of the condensed liquid (2) and (3).

The main feature of the above approach is that the description of the kinetics of condensation considers the dependence of the phase transition temperature not only on the pressure in the system, but also on the concentration of the condensing gas in the vicinity of the liquid film. Thus, yet one more condition is added to Stefan's condition — the condition of balance of the mass of the condensing gas on the film surface. In this model, the mass flow is toward the wall.

Another important feature is the fact that the parameters T_w and c_w are found on the film surface at the moment it is formed. Otherwise, we could not formulate the initial conditions to solve the problem within the framework of the unidimensional model — which are consistent with Stefan's condition for the heat flow on the film surface.

NOTATION

ρ, θ, H, u, T, average density of gas mixture, concentration of condensing component, enthalpy of mixture, longitudinal velocity component, and mixture temperature across the pipe; p, pressure in the gas flow; $\vec{\mathfrak{W}}$, gas flow velocity vector; $\mathfrak{W}_n$ and $\mathfrak{W}_\tau$, normal and tangential components of velocity vector on the phase boundary; u_w and v_w, components of velocity vector on the phase boundary; D_n, velocity of phase boundary along its normal; $\vec{q}$, vector of heat flow resulting from conduction; $\vec{j}$, vector of mass flow resulting from diffusion; q_n and j_n, normal components of heat and mass flow on the phase boundary; L, latent heat of phase transformation; p_{1w}, partial pressure of condensing component of the mixture on the phase boundary; T_* and p_*, parameters on the equilibrium phase-transformation curve; T_w, phase transition temperature; R, Boltzmann constant; m_1 and m_2, molecular weights of mixture components; p_1 and p_2, partial pressures of components in mixture; c_w, concentration of gas at the phase boundary on the side of the gas flow; ζ, α, and β, drag, mass- and heat-transfer coefficients; v, T, and c, convection rate, temperature, and concentration of condensing gas over the half-space $y > y_0(t)$; T_l, temperature of condensate in the region $0 < y < y_0(t)$; $y = y_0(t)$, coordinate of the phase boundary in the region $y > 0$; ν, $\varkappa$, $\varkappa_l$, D, kinematic viscosity of the gas diffusivity of the gas and condensate, diffusion coefficient; λ and λ_l, the thermal conductivity of gas and condensate; t, time; x and r, longitudinal and radial coordinates of a cylindrical coordinate system; y, distance from solid surface with the temperature T_b.

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DYNAMIC EQUATION OF STATE OF A GAS CONTAINING VAPORIZING DROPLETS

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On the basis of the relaxational formalism of nonequilibrium thermodynamics, a differential equation is obtained relating the pressure and volume of a gas containing vaporizing droplets. The relaxation time is calculated.

The existing methods of calculating processes of gas compression with evaporative cooling are based, as a rule, on the methods of classical thermodynamics. This leads to the necessity of assuming thermal and phase equilibrium between the droplets and the vapor–gas mixture [1-3]. With a sufficiently high rate of the process, the temperature of the vapor T and the droplet T_D will differ from the saturation temperature T_S: $T-T_S = \xi_1$, $T_D-T_S = \xi_2$. As a result, the vapor concentration will also differ from the equilibrium value: $x-x_S = \xi_3$. The rigorous description of nonequilibrium, even with fixed parameters of the cooling medium, is an extremely complex problem, and leads to very cumbersome results [4, 5]. Therefore, it is of interest to consider some simplified models of the process of nonequilibrium compression with droplets. Thus, for example, for the practical realization of evaporative cooling in compressors, it is expedient to use small concentrations of finely dispersed moisture (~30 g per kg of air) [2, 6]. The behavior of this mixture will be associated with small deviations from the equilibrium state, which may be analyzed using linear nonequilibrium thermodynamics. Below, the nonequilibrium compression of gas with droplets is analyzed on the basis of the relaxational formalism of the thermodynamics of irreversible processes.

It is assumed that the vapor, gas, and droplets, with masses M_V, M_G, and M_D, respectively, form two local-equilibrium subsystems and, in accordance with the mass-conservation law, $M_V + M_D = M_M$, the mass of injected moisture. It is expedient to introduce the notation: $\mu = M_M/M_G$, $x = M_V/M_M$, $1-x = M_D/M_V$. In accordance with the well-known ideas of [7-9], it may be assumed that, in quasisteady conditions, the intensity of heat transfer and the rate of vaporization depend on the temperature difference between the subsystems $T-T_D$. The behavior of this system will be characterized by a single effective relaxation time τ_z. Further, following [10], the operator form of the effective thermodynamic derivatives is used, allowing the adiabatic modulus k and the polytropic index n to be represented in the form of operators

$$\begin{pmatrix} \hat{k} \\ \hat{n} \end{pmatrix} = \begin{pmatrix} k_\infty \\ n_\infty \end{pmatrix} + \left[\begin{pmatrix} k_0 \\ n_0 \end{pmatrix} - \begin{pmatrix} k_\infty \\ n_\infty \end{pmatrix} \right] \left(1 + \tau_V \frac{d}{dt} \right)^{-1}. \tag{1}$$

In this expression k_∞ and n_∞ depend solely on the properties of the vapor–gas mixture (without the particles) in the initial state, and characterize the process of instantaneous compression, in which the particles do not react to the perturbation. The indices k_0 and n_0 characterize the equilibrium compression of the mixture of the gas and the droplet. It depends on the instantaneous state of the system, and varies over time. Formally repeating the derivation in [10], a dynamic equation is obtained for the adiabatic and polytropic compression processes of the vapor–gas mixture with the droplets

$$\ddot{p}V + \dot{p} \left\{ \left[\begin{pmatrix} k_\infty \\ n_\infty \end{pmatrix} + 1 \right] \dot{V} + \frac{V}{\tau_V(t)} \right\} + p \begin{pmatrix} k_\infty \\ n_\infty \end{pmatrix} \left(\ddot{V} + \frac{\dot{V}}{\tau_p(t)} \right) = 0. \tag{2}$$

In deriving this equation, it is taken into account that $k_\infty k_0^{-1} = n_\infty n_0^{-1} = \tau_p \tau_V^{-1}$; a dot over a a quantity denotes the first derivative with respect to the time; two points denotes the sec-

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